

Research in undergraduate mathematics education using the concept of didactic contract: results and contradictions?

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The concept of didactic contract is present in many studies about undergraduate mathematics education. Using a review of the literature, we explore the features of didactic contracts identified by research and their consequences for students. Papers investigating the secondary-tertiary transition observe increased expectations, and students struggling to enter a new contract. Other papers reveal contract rules conveyed by teaching practices and adopted by students, but limiting their activity. Some studies proposing interventions introducing an inquiry-oriented didactic contract observe that the ‘usual’ contract constitutes an obstacle for students’ active involvement. We discuss the apparent contradictions in the results obtained by the different categories of studies.

Keywords: Didactic contract, Expectations, Secondary-tertiary transition, Undergraduate mathematics education

Introduction

The concept of didactic contract is present in many studies about university mathematics education. At the beginning of university in particular, some of the students’ difficulties can be interpreted as resulting from a change of didactic contract, since: “Secondary school and university have different didactic contracts” (Gueudet, 2008, p. 245). But what do we actually learn from research about the features of the didactic contract(s) at undergraduate level for mathematics, and about their consequences for students? In the context of a research project aimed at uncovering mismatches between the beliefs and expectations of university teachers and those of their students, we reviewed the research literature that used the concept of didactic contract. In this paper we present this review and its outcomes. Recent studies in university mathematics education stress that the diversity of students and its consequences is an essential direction in current research (Pinto & Koichu, 2023). This diversity challenges the existence of a unique didactic contract at university; we claim that this requires to revisit the use of the concept of didactic contract in university mathematics education, and that a literature review on this theme is needed.

The didactic contract: theory

The didactic contract is one of the main concepts within the Theory of Didactical Situations (TDS, Brousseau, 1997). Many different definitions can be found in the literature, even in Brousseau’s own work. In our work we refer to the following definition:

“To each precise notion to be taught, the partners in teaching [...] associate expectations, obligations that each undertakes and benefits from, and the means by which they envisage (mutually or separately) satisfying these expectations and obligations as well as the consequences of not satisfying

them. A didactic contract is, in the broad sense, an interpretation of the set of these expectations and obligations, be they compatible, explicit, and agreed to or not.” (Brousseau & Warfield, 2014, p.1)

The concept of didactic contract is complex; it has been the subject of debates in the French community of research in didactics of mathematics (Sarrazy, 1995). We do not intend here to propose a comprehensive theoretical discussion, but rather to foreground some issues relevant in our context. A first aspect concerns the implicitness of the didactic contract’s rules: most of these rules are implicit, and it is neither possible nor desirable to present them explicitly to the students (González-Martín et al., 2014). This raises a methodological issue for identifying the rules of the didactic contract. A second issue concerns what might be called the “level of generality” of the didactic contract. The definition cited above mentions a didactic contract for “each precise notion”; some studies identify didactic contract rules for specific mathematical situations. Chevallard (1988) claims that entering an institution involves a subjection to an institutional contract, which includes a didactic contract for the mathematical contents living in this institution. Balacheff (1988) proposes to distinguish between the didactic contract, corresponding to interactions between a teacher and their students in a situation, and what he calls “custom,” which is a set of implicit but more general rules shared in a given classroom for example. This debate concerns the level of generality of the didactic contract, but also a vision of the contract as a set of already existing, stable rules, versus a dynamic process of negotiating the contract along the interactions between the students and the teacher. The research community has not adopted the concept of custom, while at the same time an increasing number of studies has used the concept of didactic contract to study a broad range of implicit features of the teaching and learning processes. De Vleeschouwer and Gueudet (2011) propose to distinguish between three levels of contract in a given institution: a general level, independent of the knowledge at stake ; the level of mathematics as a discipline; and the level of particular mathematical notions.

Drawing on these theoretical elements, we investigate the following research questions:

Which features of didactic contract(s) at the three different levels are identified by research in mathematics education, with which consequences for students? What categories emerge from a grouping of papers according to these features and consequences?

Review of the literature : methods

To answer our research questions, we conducted a review of the research literature in university mathematics education. We were interested in international research using the concept of the didactic contract to analyse teaching and learning features at the undergraduate level. We focused on research conducted in the last twenty years, i.e. research published between 2005 and 2024. We examined books and book chapters concerning university mathematics education; international journals (IJRUME, ESM, ZDM, IJMEST), and the proceedings of the CERME, EMF and INDRUM conferences. We added to this first selection some of the papers cited in it.

We have included in this review only works where the concept of the didactic contract plays an important role. These comprise either synthesis papers that emphasize the concept of the didactic contract, or empirical studies that use this concept in their data analysis methodology and/or present results in terms of didactic contract. We reviewed a total of 40 papers. In each of the selected papers,

we noted the research questions investigated; we examined the features of the didactic contract(s) identified (contract rules in particular, if such rules are explicitly mentioned), and their consequences for students. Moreover, for each study we identified the level(s) of the contract concerned: general (independent of knowledge), disciplinary (related to mathematics) or content-specific (such as calculus or linear algebra).

Results

Three main categories emerged from our reading of the papers; Table 1 shows these categories and the number of papers in each. The length constraints led us to discuss in this text only some selected papers that represent each category.

Table 1. The three categories, and the number of papers in each

Changes of the contract during the secondary-tertiary transition	The ‘usual’ university contract, adopted but limiting students’ mathematical activity	Alternative didactic contract, accepted or not by students	Other	Total
14	8	14	4	40

Investigating changes of the didactic contract during the secondary-tertiary transition

A first type of studies concerns the secondary-tertiary transition, the change of didactic contract during this transition, and the difficulties of novice students to enter the university contract.

De Vleeschouwer and Gueudet (2011) observe through a university textbooks analysis didactic contract rules concerning the teaching and learning of duality in linear algebra (content level). In particular, since a linear form is sometimes considered as a function and sometimes as an element of a vector space, students should be able to recognise which point of view is relevant and should be flexible in using either view; this flexibility is typical from university expectations, compared with secondary school. Based on this finding, the authors design an intervention in which a systematic work on this aspect of the contract is organised. The analysis of students’ answers to a questionnaire shows that this teaching helped them to develop this contract rule. We note that in this work, an intervention is designed to support the adoption by students of some rules of the university contract.

Studies concerning proof (and thus at the discipline level), synthesized by Selden (2012) show that the didactic contract changes between secondary school and university with increasing expectations, especially in terms of rigour, and that novice students have difficulties linked with this new contract. Bloch and Ghedamsi (2005) study the secondary-tertiary transition concerning calculus. They introduce ten “macro-didactic” variables that characterise the “actual didactic contract”, for example: “The degree of formalization”; “The quantity of new notions introduced”; “The status of the tasks proposed: technical application or proof of a general result”. The analysis of secondary school textbooks and of lists of exercises used at university (in Tunisia) evidences a change in the values of the ten variables and characterises the didactic contract for calculus in secondary school and at university. Even if used here for a calculus course, these variables can characterise the didactic contract at a disciplinary level; they reveal increased expectations at the beginning of university.

Pepin (2014) investigates students' experience during the secondary-tertiary transition, interpreted as their entrance in a new didactic contract. Referring to the three levels introduced by de Vleeschouwer and Gueudet (2011), she studies in particular the general and the disciplinary levels. Drawing on interviews with students over two years, interviews with teachers and classroom observations, she identifies changes experienced by the students. In secondary school, the students are strongly supported by the teachers, while at university they are expected to find their own learning strategies. Combining the didactic contract and a resource perspective, Gueudet and Pepin (2018) further observe that "discussions with the teacher are not easily available resources for students" (Gueudet & Pepin 2018, p.62) is a general contract rule at university; as a consequence, students develop a rule such as "in case of difficulties, I can find help from my peers" (Gueudet & Pepin 2018, p. 69). At the disciplinary level, there is a mismatch between the teacher's views on the homework to be done (including a work on the text of the lecture) and the actual homework done by the students (focused on exercises, and abandoning the search when the solution is not found quickly). Gueudet and Pepin (2018) also observe that the use of online resources can lead some students to incorporate irrelevant contract rules (here at the level of the content, concerning number theory).

Didactic contract rules limiting students' activity

A second type of studies concerns contract rules conveyed by teaching practices at university, of which the teachers may be unaware, and whose consequence is a limitation of students' mathematical activity. Lebaud (2010) investigates how teachers design mathematics exam texts by using interviews and analysing successive versions of the exam text. She finds that teachers' choices are influenced by didactic contract rules, such as: "The values to be calculated when solving an equation with complex numbers are integers" (content level) or "The answer to the first question of an exercise can appear in the text of the second question" (disciplinary level). González-Martín (2013) investigates the teaching and learning of series. He identifies contract rules through a textbooks analysis. Since the teachers align with the textbooks' content, he hypothesises that the contract rules he identified influence the students' activity. By proposing purposefully designed tasks, he confirms that students have integrated two selected implicit rules (content level): "To solve the questions about series that are given, their definition is not necessary," and "Applications of series, inside or outside of mathematics, are not important" (González-Martín, 2013, p.2328). Dorko (2020) analyses the activity of students using an online homework system proposing exercises about sequences. She identifies that one of the students' strategies is to draw on didactic contract rules at the level of the content, like "when a list of sequences is proposed, some are convergent and some are divergent." The results obtained in such studies reveal implicit contract rules that are conveyed by certain features of teaching and of which the teacher is unaware; González-Martín (2013) describes them as differences between the knowledge to be taught and the knowledge actually taught. Some students leverage contract rules to provide expected answers, instead of reasoning mathematically; moreover some rules limit what is actually expected of students.

Introducing an alternative didactic contract: is it possible?

Most intervention studies using the concept of didactic contract concern inquiry-oriented courses, conveying a specific didactic contract, different from the didactic contract in 'traditional' courses.

We use below the term 'usual didactic contract', which appears in some of these articles, but is not clearly defined by the authors. Grønbæk et al. (2009) study a project-based course, where students work in teams. The authors describe rules of the 'traditional' didactic contract at the general level, e.g. the students have to "do the assigned homework"; the teacher has to "institutionalise good solutions and answer students' questions." In the project-based course, these rules are broken and the teams of students organise their work autonomously. Following two teams of students, the authors observe that what they call an "a-didactic contract" develops in the teams, with rules about the important tasks to be performed and how they are shared within the group. This contract has a positive impact on students' representations about the problem-solving activity. Barquero et al. (2022) note similar changes in the didactic contract, in the context of Study and Research Paths (SRPs: courses starting from an open question where teams of students build their own paths in terms of sub-questions and associated answers). Students have more responsibilities, such as for planning their work or validating an answer. Teachers are no longer the sole holders of knowledge in the classroom. The authors interpret this new contract as a paradigm shift, from 'visiting monuments' to 'questioning the world'. Nevertheless this new contract, very different from the 'usual' one, has to be established in the class and its negotiation can be a delicate process. Dawkins et al. (2019) ask whether or not students adopt the new rules of the didactic contract in an inquiry-based course. Interestingly, they note that some students adhere to the new rules, while others reject them, and this is related to their orientation towards learning mathematics: learning orientation (high buy-in) vs performance orientation (low buy-in).

The study by Ellis et al. (2025) does not concern an intervention; but it evidences the possibility of negotiating different contracts. The authors study the link between students' homework and the didactic contract in the context of a national survey in the US involving 500 institutions. They select 5 universities with successful Calculus I programs and, based on a rich set of data (classroom observations and videos, students' productions, interviews with students and teachers), they observe that the didactic contract regarding homework (at the general and disciplinary levels) in these universities differs from the contract in the other universities. Students are expected to struggle with a more complex content; but they do not struggle alone, they are provided with relevant material to support them in their homework; they are also expected to explain their thinking, and they receive feedback. This study evidences the existence of different didactic contracts in different universities, and the possibility of installing a didactic contract fostering a productive homework.

The papers coded "Other" address specific issues like didactic contract rules when using digital resources, like videos or online exercises. The study by Gill and O'Donoghue (2007) concerns the didactic contract in mathematics service courses in Ireland. Interestingly, the authors identify mismatches between students' expectations and the course delivery. Nardi (2008), interviewing lecturers, notes contradictory expectations which lead her to coin the expression "fuzzy didactic contract", and to expect difficulties for students to understand the rules of this contract.

Discussion

We identified three main categories of papers: (1) papers focusing on the change of didactic contract during the secondary-tertiary transition; (2) papers identifying limitations in the students'

mathematical activity resulting from didactic contract rules; (3) intervention studies proposing an “alternative contract.” The features of the didactic contract identified by the studies depend on the category. In category (1), the authors note an increase in expectations of students (e.g. in terms of flexibility, rigor, autonomy). This change creates difficulties for students. Moreover, they are sometimes unaware of this change in expectations; making the new rules explicit can support their entrance in the new contract. The studies in category (2) unveil contract rules conveyed by the teaching practices at university, of which the teachers are unaware. These rules can prevent students from developing a rich mathematical activity. Finally, in the context of research-based interventions, studies in category (3) focus on a didactic contract that is different from the ‘usual’ contract at university. For such studies, the ‘usual’ contract can constitute an obstacle, since some students reject the breaches in the rules introduced by the new contract of the intervention.

These answers to our research questions raise some issues, since some of their results can seem contradictory. In the context of an ‘ordinary’ teaching, studies in category 1 evidence significant expectations at university, while category (2) studies identify limitations in what is actually expected at university, at the level of the discipline and of the content. This apparent contradiction could mean that the expectations and obligations of students at university, although higher than those at secondary school, remain below possible expectations for the mathematics taught. A second issue concerns the entrance of students in the didactic contract: while category (1) studies foreground the difficulty for students to enter into the new contract, studies of the second and third categories, on the opposite, produce results showing that students adopt this ‘usual’ contract. Moreover this adoption has negative consequences: limitation of the students’ mathematical activity, or obstacle for their involvement in an inquiry-oriented teaching. Would the students adopt the ‘usual’ didactic contract only when researchers want them to act differently? An explanation could be that most students, within a few months, actually comply with rules conveyed by the teaching practices shared in a majority of courses (the ‘usual’ contract), and that breaking these rules is complex. This leads to a third issue: the existence of a ‘usual’, dominant contract, regardless of the university, the teacher and the students. Intervention studies proposing an inquiry-oriented didactic contract evidence the possibility, under specific conditions, of negotiating alternative contracts. Even in ‘ordinary’ conditions, students who have several mathematics teachers at university may experience different contracts. The same teacher may also have different kinds of interactions with different groups of students, and this can produce several contracts in the same class. Elucidating all these issues and apparent contradictions requires further research.

Conclusion

While many studies in university mathematics education refer to the concept of didactic contract, there is still a lot to be learnt about didactic contracts at university. Pursuing in our project the aim of determining implicit expectations of teachers and students, we will not restrict ourselves to the didactic contract but will also consider other theories likely to enlighten such implicit expectations, for example the commognitive theory (e.g. Sfard, 2023). Moreover so far the concept of didactic contract is not associated with a solid method allowing to identify its rules; considering other theories can suggest principles for such a method. Understanding the mismatches between students and teachers at university probably requires a combination of several theories.

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